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A TWO-DIMENSIONAL, TIME-DEPENDENT, COMPUTER STUDY OF THE LOW-VOLTAGE,
PARALLEL-PLANE DIODE

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I. INTRODUCTION

The low-voltage, parallel-plane diode has been studied as a computer code check preliminary to a full relativistic treatment which is of ultimate interest. A one-dimensional, time-dependent study has been reported elsewhere.⁽¹⁾ The next step in this development will be the addition of self magnetic fields at moderate energies before including, finally, the relativistic equations of motion.

II. GREEN'S FUNCTION

The method employed to take account of the particle interactions is the particle-in-cell approximation.⁽²⁾ One must calculate, therefore, a table of Green's functions for each mesh point, each table entry being due to an assumed unit charge in each cell. In the two-dimensional cylindrically symmetric case, we are dealing with rings of charge. If we consider the "walls" of the diode to be infinitely far away we can calculate the inter ring force for free rings. We then take into account the conducting planes at anode and cathode by the method of images.

Consider the potential at r, z of a uniformly charged ring, radius r' at z' (see figure 1).

$$\Phi(r, z) = \frac{Q}{8\pi^2 \epsilon_0} \int \frac{d\phi'}{\sqrt{r'^2 + a^2 - 2r'a \cos\psi}}$$

where $a = \sqrt{r^2 + z^2}$ and $\cos\psi = \cos\phi' \cos\phi$.

But, $\phi = \cos^{-1}(r/\sqrt{z^2 + r^2})$, so

$$\Phi(r, z) = \frac{Q}{8\pi^2 \epsilon_0} \int_0^{2\pi} \frac{d\phi'}{\sqrt{r'^2 + z^2 + r^2 - 2rr' \cos\phi'}}$$

Let $\alpha^2 = 4rr'/((z-z')^2 + (r+r')^2)$. Then,

$$\Phi = \frac{-2Q\alpha}{8\pi^2 \sqrt{rr'} \epsilon_0} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \alpha^2 \sin^2 \phi}}$$

Thus

$$E_r = -\frac{\partial\Phi}{\partial r} = \frac{Q}{4\pi^2 \epsilon_0 r \sqrt{(r+r')^2 + (z-z')^2}} \left\{ K(\alpha) - \frac{[r'^2 - r^2 + (z-z')^2]}{(r-r')^2 + (z-z')^2} E(\alpha) \right\}$$

and

$$E_z = -\frac{\partial\Phi}{\partial z} = \frac{Q(z-z') E(\alpha)}{2\pi^2 \epsilon_0 \sqrt{(r+r')^2 + (z-z')^2} ((r-r')^2 + (z-z')^2)}$$

where $K(\alpha)$ and $E(\alpha)$ are the complete elliptic integrals of the first and second kind, respectively

$$K(\alpha) = \int_0^{\pi/2} (1 - \alpha^2 \sin^2 \theta)^{-\frac{1}{2}} d\theta,$$

$$E(\alpha) = \int_0^{\pi/2} (1 - \alpha^2 \sin^2 \theta)^{\frac{1}{2}} d\theta.$$

These are approximated as follows⁽³⁾. Let $\beta = 1 - \alpha^2$, $\gamma = \alpha^2 \log \beta$.

Then

$$K(\alpha) \cong 1.38629436 - 0.5\gamma + \beta(0.1119697 - 0.1213486\gamma + \beta(0.07253230 - 0.02887472\gamma)).$$

and

$$E(\alpha) \cong 1.0 + \beta(0.4630106 - 0.2452740\gamma + \beta(0.1077857 - 0.04125321\gamma)).$$

To take account of the conducting planes at cathode and anode we must superimpose a series of charges to give zero potential at these planes. One finds that, assuming a negatively charged ring as the primary source at z_s , there will be positive images at $2nD - z_s$ and negative images at $2nD + z_s$ where D is the cathode-anode spacing and n runs through all integers, positive, negative, and zero.

We ran some tests for various combinations of source point and field point to determine how many images are necessary to adequately determine the fields. The results are given graphically in figure 2 and indicate that 20 images are sufficient.

III. INITIAL CONDITIONS AND THE INJECTION SIMULATION

In order to simulate a hot cathode we give the injected rings a temperature as follows. We assume a Maxwellian distribution in v_r and v_z

$$f(v) = \frac{mv}{kT_e} \exp\left(-\frac{mv^2}{2kT_e}\right).$$

We have at our disposal a random function generator, RANF, which gives, by repeated calls, a set of numbers $\{R_i\}$ uniformly distributed between zero and 1. In order to transform this set to a set distributed according to $f(v)$ we form the cumulative distribution function⁽⁴⁾

$$F(v) = \int_0^v f(v) dv$$

which gives the probability that a ring have velocity v or less. We can specify the ring velocities by solving

$$R_i = F(v_i)$$

for v_i . Reference to figure 3 indicates how a set of uniformly spaced points on the y axis projected through the curve $F(v)$ gives a set of points on the x axis distributed according to $f(v)$. Attention must be paid to the fact that, whereas the r velocities can be negative and positive, the z velocities are positive only.

A similar approach is used to insure a constant-density emitting area. Note that we have specified that the rings have equal charges. Thus, if we were to give a uniform random distribution of rings at the cathode we would have in fact specified a beam with a high-density core. Thus, we want

$$f(r) = 1 \quad 0 \leq r \leq 1$$

$$\frac{1}{\pi} \int_0^{2\pi} \int_0^r f(r) r dr d\phi = r^2 \equiv F(r)$$

where we have normalized so that $r=1$ when the random number generator produces $R_i=1$. Figure 4 graphically indicates the solution of $R_i = F(r_i)$ for the initial radial coordinates r_i .

Although we inject a fixed number of rings per iteration and the iterations are separated by a fixed time difference, we can, nevertheless, simulate random injection times as follows.⁽⁴⁾ We specify the average number of electrons, $n_{\Delta t}$, emitted in the time interval, Δt , between iterations. The probability that s electrons will be emitted in Δt is given by the Poisson distribution

$$f(s) = \frac{e^{-n_{\Delta t}} (n_{\Delta t})^s}{s!}$$

The cumulative distribution function is then

$$F(s) = \sum_{t=0}^s f(t).$$

Now, to simulate random injection we generate a random number R and compare it with $F(s)$. If $F(s'-1) \leq R < F(s')$ we generate s' additional random numbers $\{R'_i\}$. We would like to take the injection times to be at $R'_i \Delta t$, ($i=1, \dots, s'$). But, for simplicity, we assume all particles injected at the same time but at varying small distances in front of the cathode given by $z_i = R'_i \Delta t v_i$ where v_i is the initial velocity of the i th electron.

IV. ELECTROSTATIC FOCUSING

The beam being studied is a finite cylinder of charge which expands along its length due to space charge repulsion. This expansion makes it difficult to compare quantitatively with the theory of the parallel-plane diode. There exists a method⁽⁵⁾, however, by which one can force the finite beam into laminar flow perpendicular to the electrodes. This gives the effect of an infinitely wide beam.

It is well known⁽⁶⁾ that the potential variation with distance in a planar diode is given by

$$V = Az^{4/3}$$

where A depends on the anode current. One also has

$$\frac{\partial V}{\partial r} = 0$$

everywhere.

The desired potential distribution external to the beam is found from Laplace's equation. If an analytic solution can be found, then the real and imaginary parts of this solution are also solutions. Thus,

let $z = z+ir$ (r measured from beam edge) so we have

$$\phi+i\psi = A(z+ir)^{4/3} .$$

Hence

$$\phi = A(z^2+r^2)^{2/3} \cos \left[\frac{4}{3} \tan^{-1} \frac{r}{z} \right] .$$

Thus there is a zero potential along a line passing through $z=0$ and making an angle $\frac{3}{4}(\pi/2) = 67.5^\circ$ with the beam edge. If an electrode at cathode potential is placed along this equipotential then the fields external to the beam will be such that the electrons will flow as if they were part of an infinite laminar beam. This is called the "Pierce Electrode".⁽⁵⁾

The existence of such an electrode is accommodated in two ways. First we note from figure 5 that if a ring arrives in region II an image will be produced in the Pierce electrode. The image of this image will be produced in the anode. Furthermore, if the ring also has $r > b$ (not likely if the Pierce electrode works) we no longer have to take into account the infinite sequence of images in anode and cathode.

A more profound effect of the electrode is the distortion of the field lines from their formally rectilinear array. Consider the polar coordinate system in figure 6. Suppose the potential at the point R , θ is

$$v = R^p \sin p\theta$$

which is the correct conformal mapping for our problem if we set $p = \pi / (\theta_0 + \pi/2)$, for then $u=0$ at $\theta=0$ and $\theta=\theta_0 + \pi/2$. The direction of the field at the point R , θ is then

$$\phi = \tan^{-1} \left(\frac{\partial v}{\partial r} / \frac{\partial v}{\partial z} \right) = \frac{z \cos p\theta - (b-r) \sin p\theta}{z \sin p\theta + (b-r) \cos p\theta} .$$

Along the field lines we have $|E| = -V_0/D$. Hence

$$E_r = \left(-V_0/D \right) \sin \phi$$

and

$$E_z = \left(-V_0/D \right) \cos\phi.$$

This is true near the cathode. Near the anode the fields are still rectilinear. We add the curvature in ever increasing amounts as we pass from anode to cathode.

V. RESULTS

Figure 7 is a plot of the ring positions after equilibrium sets in for the case of a non-zero temperature and no Pierce electrode. Figure 8 indicates the result of installing a Pierce electrode. In order to better compare with Langmuir-Child theory we set the r-temperature to zero while leaving the z-temperature non zero, and the flow is shown in figure 9. The plot of potential versus distance is shown in figure 10 and is compared with the Langmuir-Child formula. Figure 11 is a higher-temperature plot to show the position of the potential minimum more clearly. Figure 12 shows the anode current versus time and compares the equilibrium value with the Langmuir-Child value. Figure 13 is a graph of the total number of rings in the system at any time. The equilibrium is maintained by removing those rings which are turned back to the cathode and those which reach the anode. The Pierce electrode drives all rings away from the extreme edge (in the r-direction) of the mesh so all particles passing through the potential minimum eventually make it to the anode.

Table I gives the parameters of the problem. The appendix contains the program listings.

TABLE I - PARAMETERS

Anode Potential	10 Volts
Thermal Potential	0.1 and 0.5 Volts
Charge per Ring	-1.6×10^{-17} Coulombs
Mass per Ring	9.1×10^{-29} Kilograms
Anode-Cathode Spacing	1.852×10^{-3} cm
Emission Radius	1.32×10^{-3} cm
Maximum Number of Particles Available	1500
Number of Cells: r direction	7
z direction	7
Time Step per Iteration	2×10^{-12} seconds
Angle of Pierce Electrode	67.5° and 90° (no electrode)
Average Number of Rings Injected per Time Step	50

REFERENCES

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2. F. H. Harlow and W. M. Evans, Los Alamos Report, LA2139 (1957).
3. W. J. Cody, Mathematics of Computation, 19, Number 89, 105 (1965).
4. P. K. Tien and J. Moshman, JAP, 27, 1067, (1956).
5. J. R. Pierce, Theory and Design of Electron Beams, D. Van Nostrand, 1954, p. 179f.
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FIGURE CAPTIONS

- Figure 1 Geometry for calculating field at (r, z) due to ring of charge, radius r' , at z' .
- Figure 2 Plots of E_r and E_z for 5 different dispositions of ring source and field point versus number of images in cathode and anode plane.
- Figure 3 Cumulative distribution function for Maxwellian velocity distribution (from Tien and Moshman, JAP 27, 1067, (1956) Fig. 10).
- Figure 4 Cumulative distribution function for uniform distribution of emitted current due to rings of equal charge.
- Figure 5 Image forces in Pierce electrode for rings for which $\tan^{-1} b-rs/zs < \pi/2 - \theta_0$.
- Figure 6 Geometry for calculating field direction in neighborhood of cathode with a Pierce electrode.
- Figure 7 Plot of ring positions for case of r-temperature of 0.1 Volts and no Pierce electrode.
- Figure 8 Plot of ring positions for case of r-temperature of 0.1 Volts with Pierce electrode at 67.5° with respect to beam edge.
- Figure 9 Plot of ring positions for case of zero r-temperature with Pierce electrode at 67.5° with respect to beam edge.
- Figure 10 Plot of $V(z)$ versus z showing depression of potential due to virtual cathode formation. Temperature = 0.1 Volts.
- Figure 11 Plot of $V(z)$ versus z for high temperature ($V_T = 0.5$ Volts) bringing virtual cathode out near first mesh point.
- Figure 12 Plot of anode current versus time and comparison with temperature corrected Langmuir-Child value.
- Figure 13 Plot of number of rings in diode versus time showing equilibrium between those injected at cathode and those lost to anode or returned to cathode by space-charge cloud.

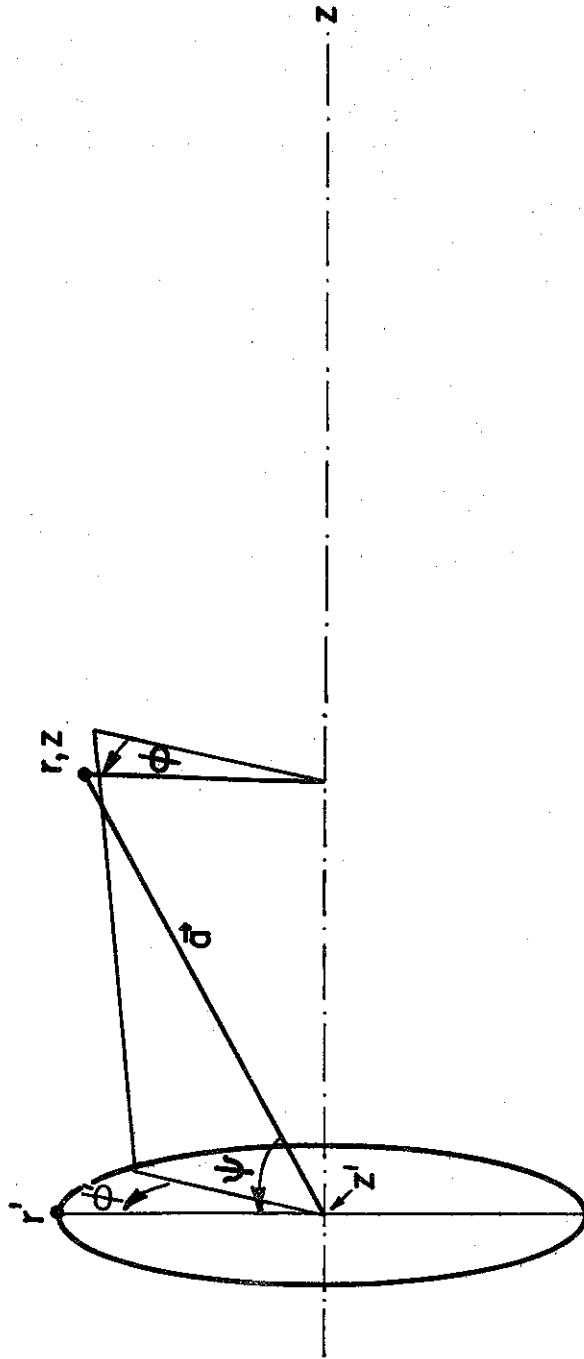


Figure 1

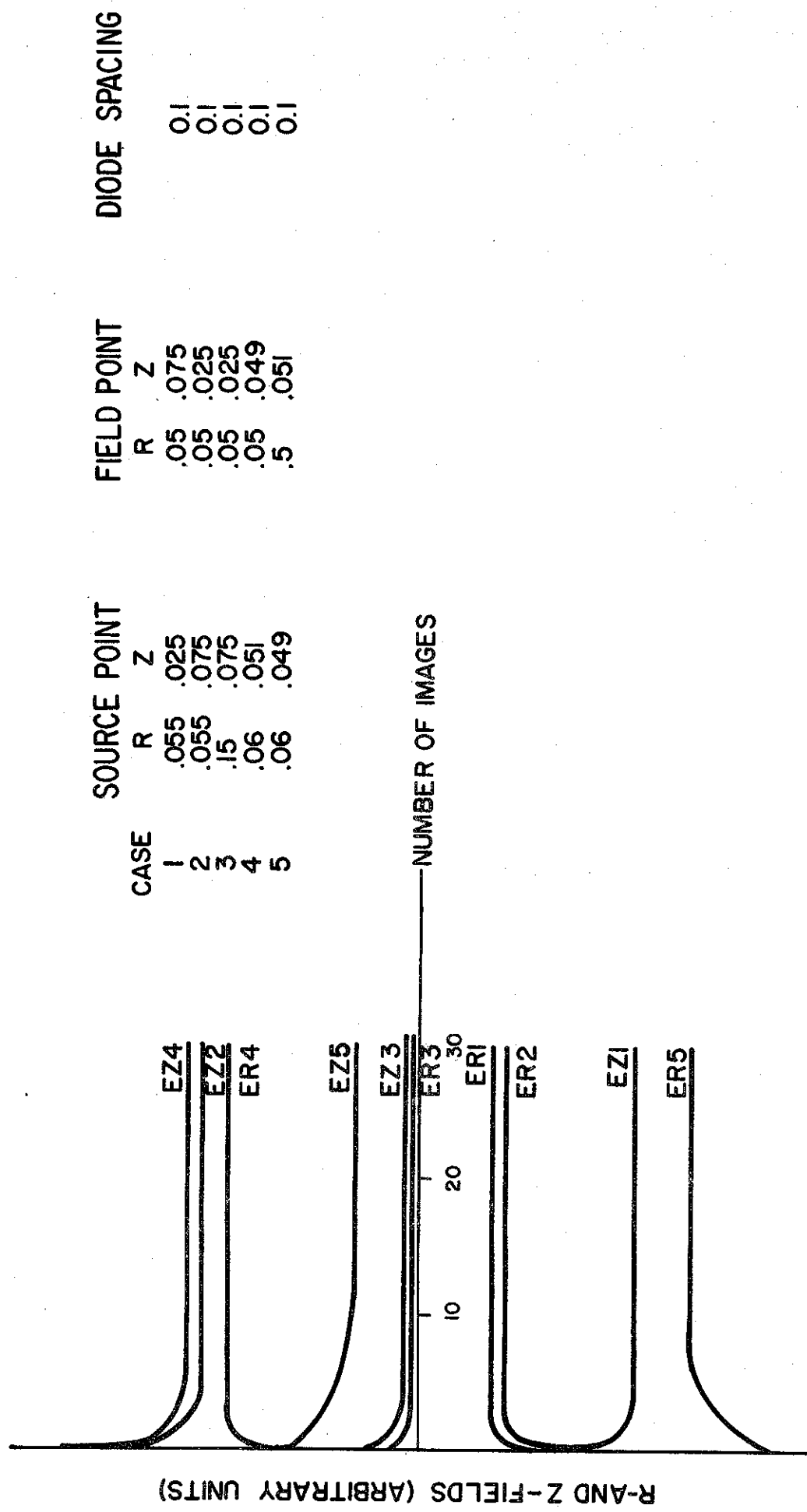


Figure 2

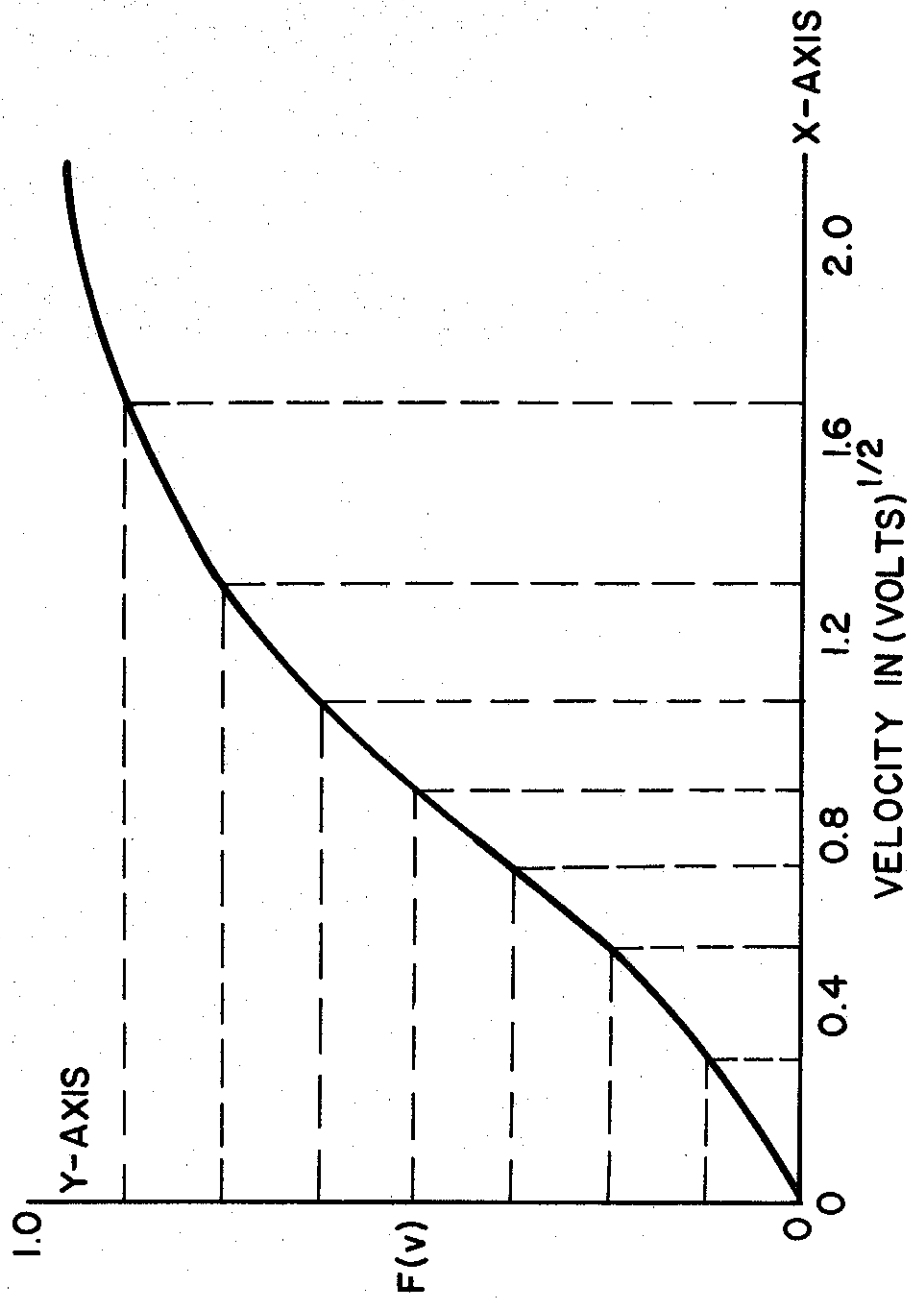


Figure 3

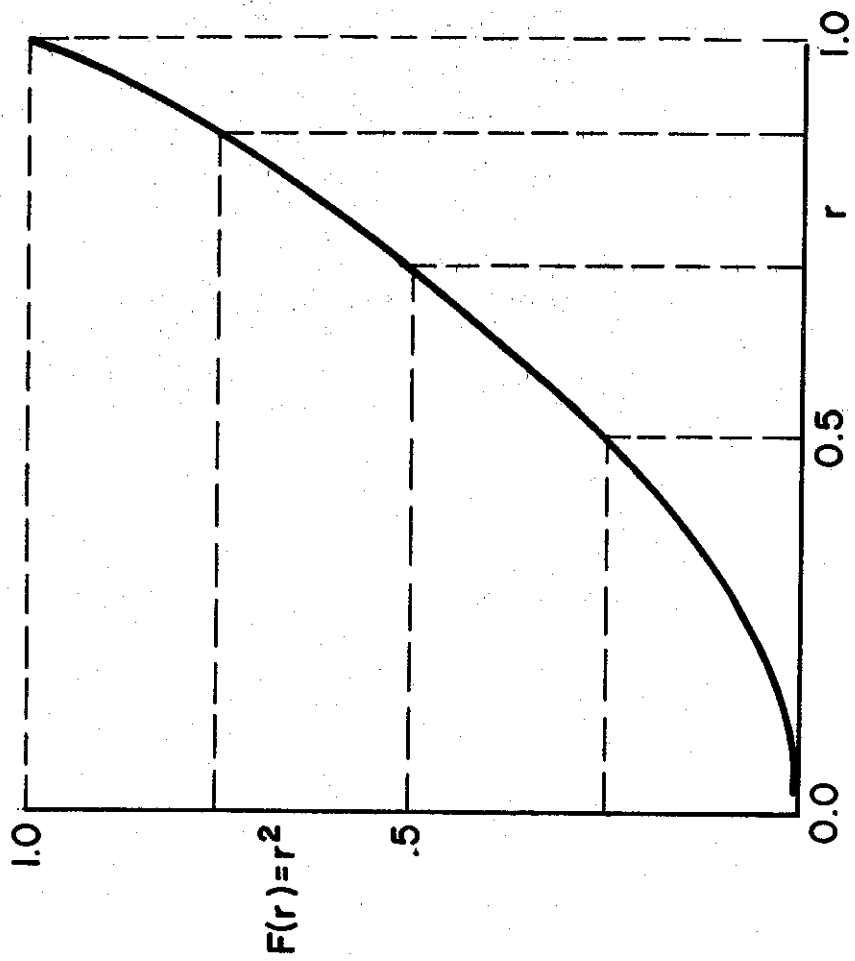
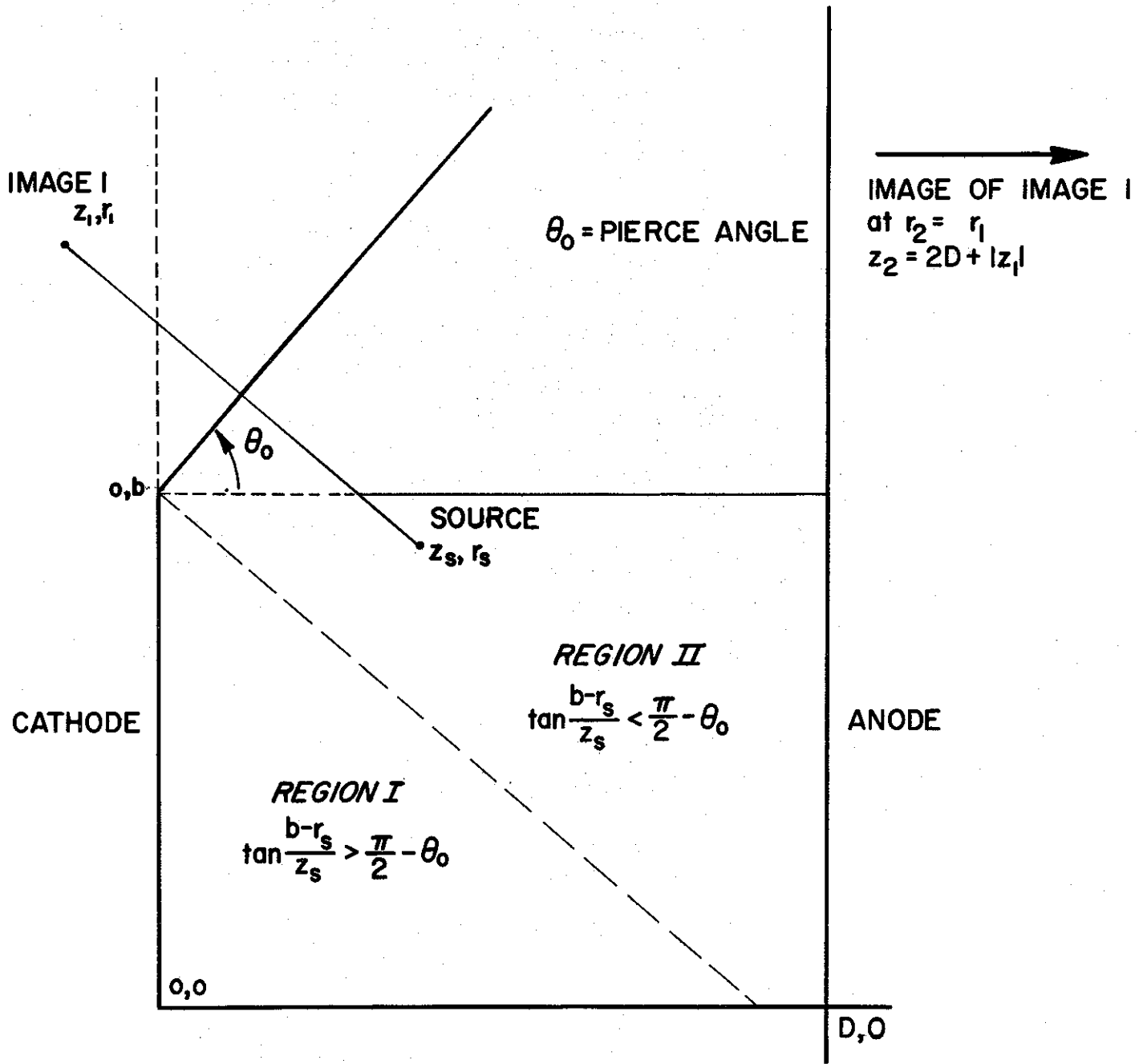


Figure 4



RING AT: r_s, z_s

IMAGE AT: $r_1 = 2d \sin(\pi/2 - \theta_0) + r_s$
 $|z_1| = 2d \cos(\pi/2 - \theta_0) - z_s$

WHERE $d = \sqrt{(b-r_s)^2 + z_s^2} \sin \left[\theta_0 + \tan^{-1} \left(\frac{b-r_s}{z_s} \right) \right]$

Figure 5

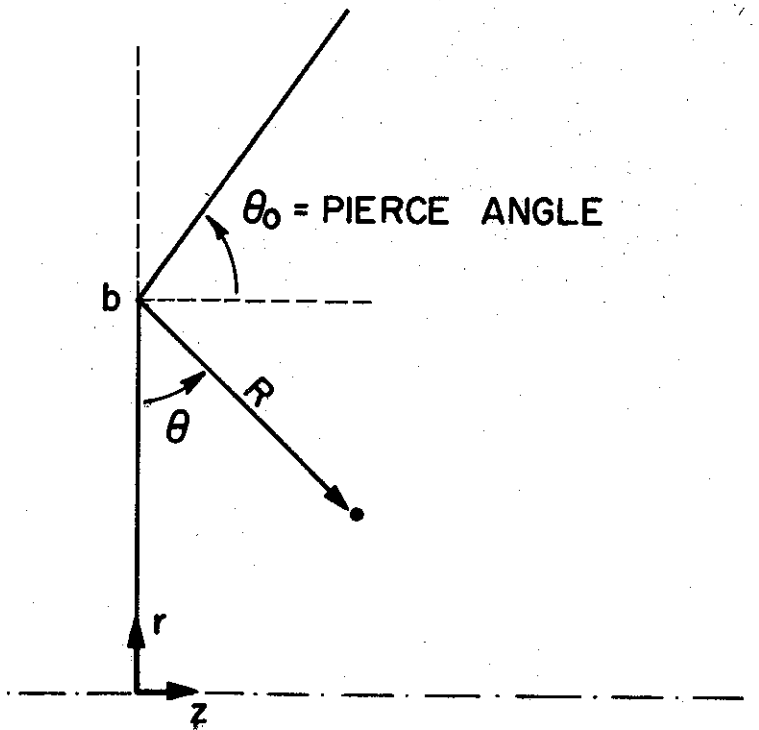


Figure 6

PARTICLE POSITIONS AT TIME $T = 7.80000 \times 10^{-11}$

0.0000+000

5.0000-006

1.0000-005

1.5000-005

2.0000-005

5.0000-005

4.0000-005

3.0000-005

2.0000-005

1.0000-005

-1.3323-015

NO PIERCE ELECTRODE
TEMPERATURE = 0.1 VOLTS

$V_0 = 10$ VOLTS

ANODE

CATHODE

Figure 7

PARTICLE POSITIONS AT TIME $T = 7.80000 \times 10^{-11}$

0,0000+000

5,0000-005

1,0000-005

1,5000-005

2,0000-005

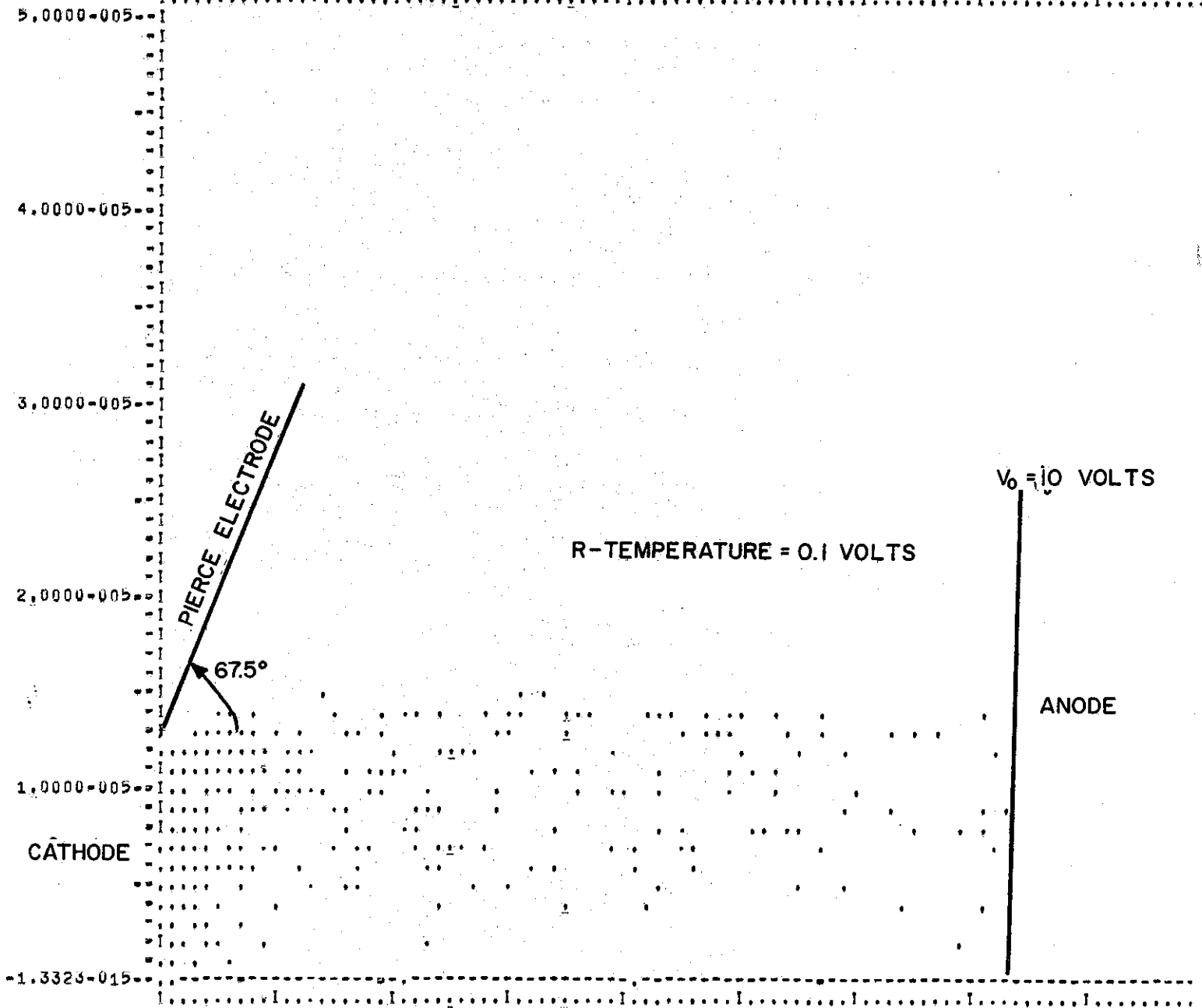


Figure 8

PARTICLE POSITIONS AT TIME $T = 4.6000 \times 10^{-11}$

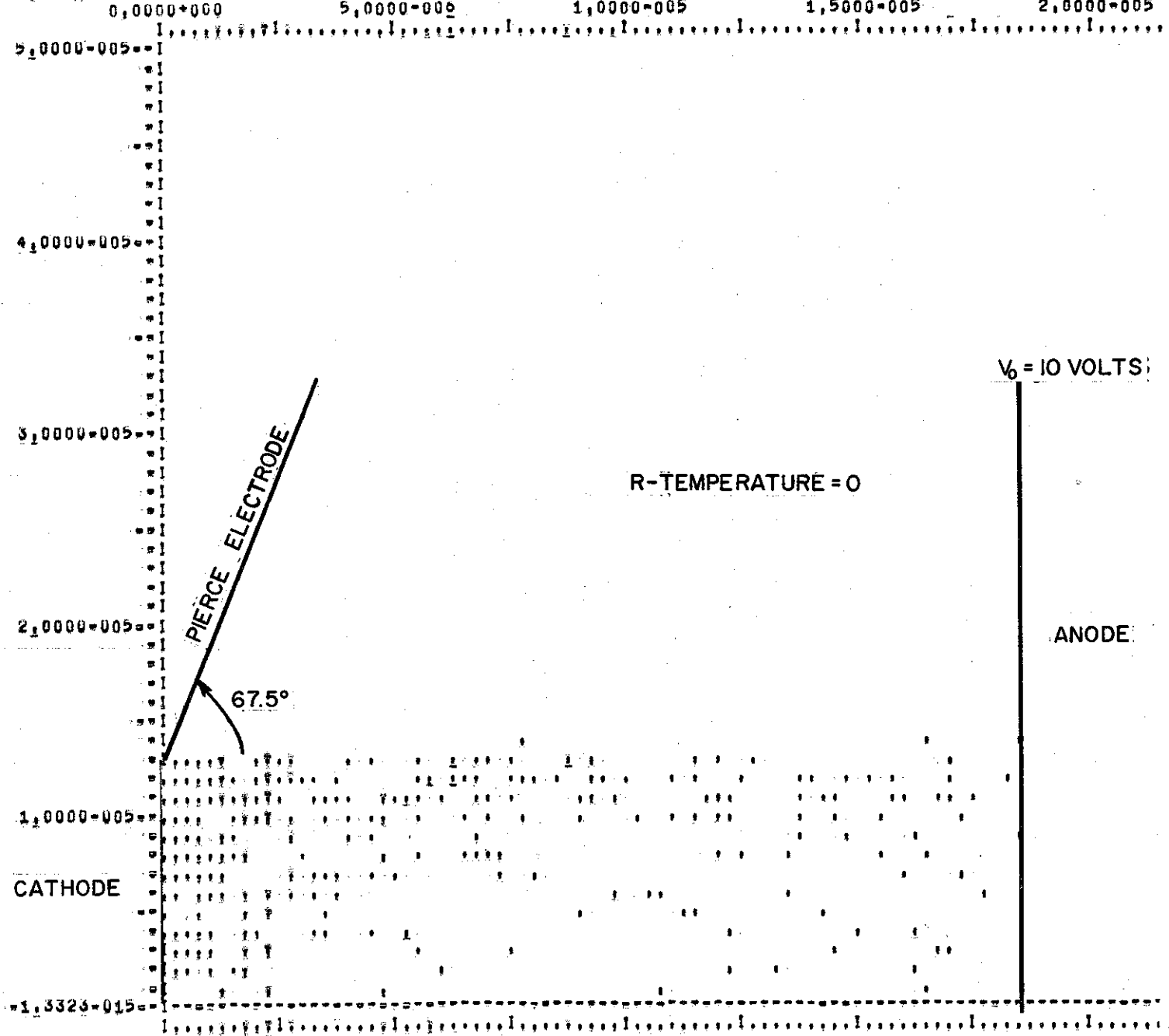


FIGURE 9

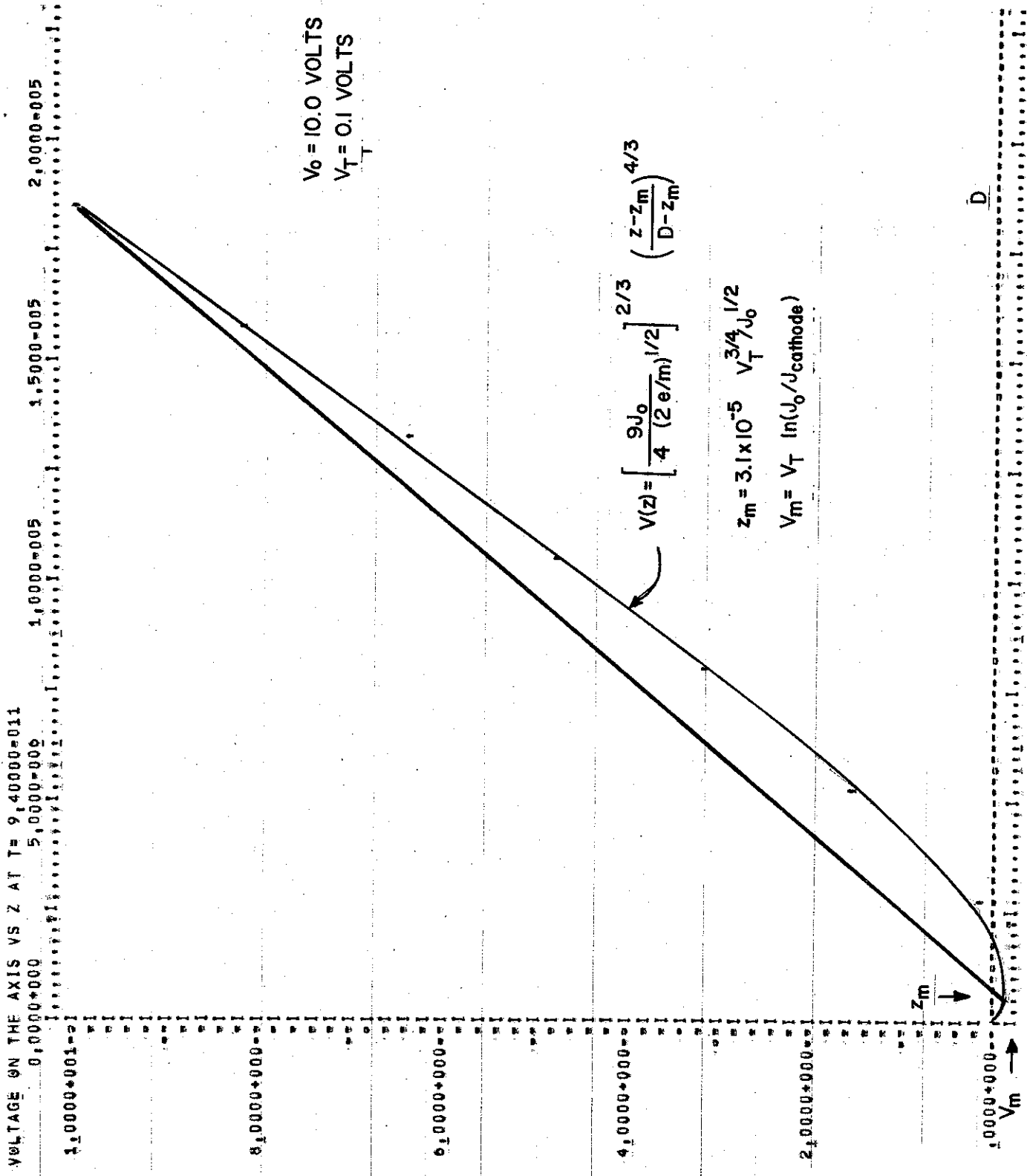


FIGURE 10

VOLTAGE ON THE AXIS VS Z AT T = 9.40000-011

0.0000+000 5.0000-006 1.0000-005 1.5000-005 2.0000-005

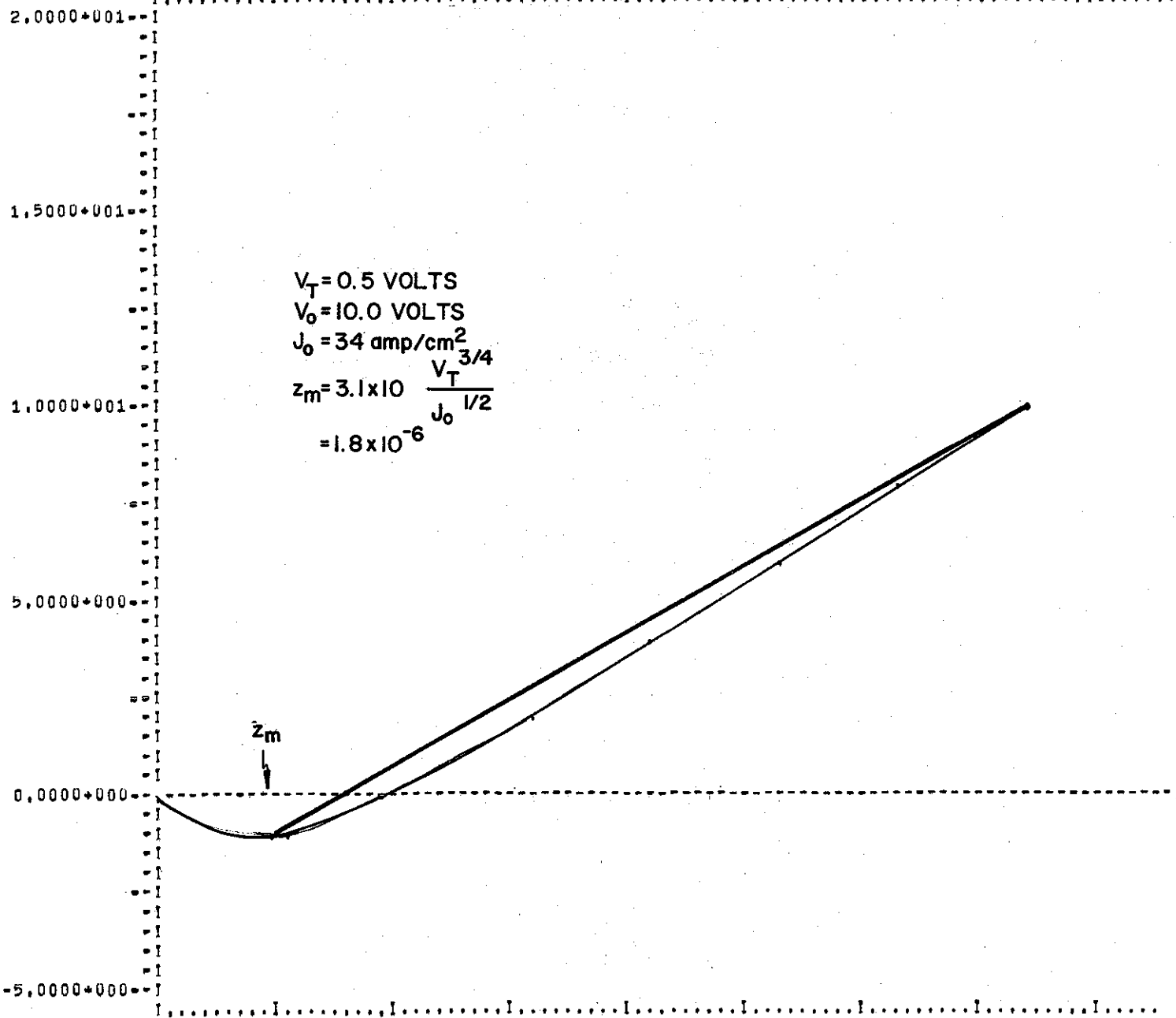


FIGURE 11

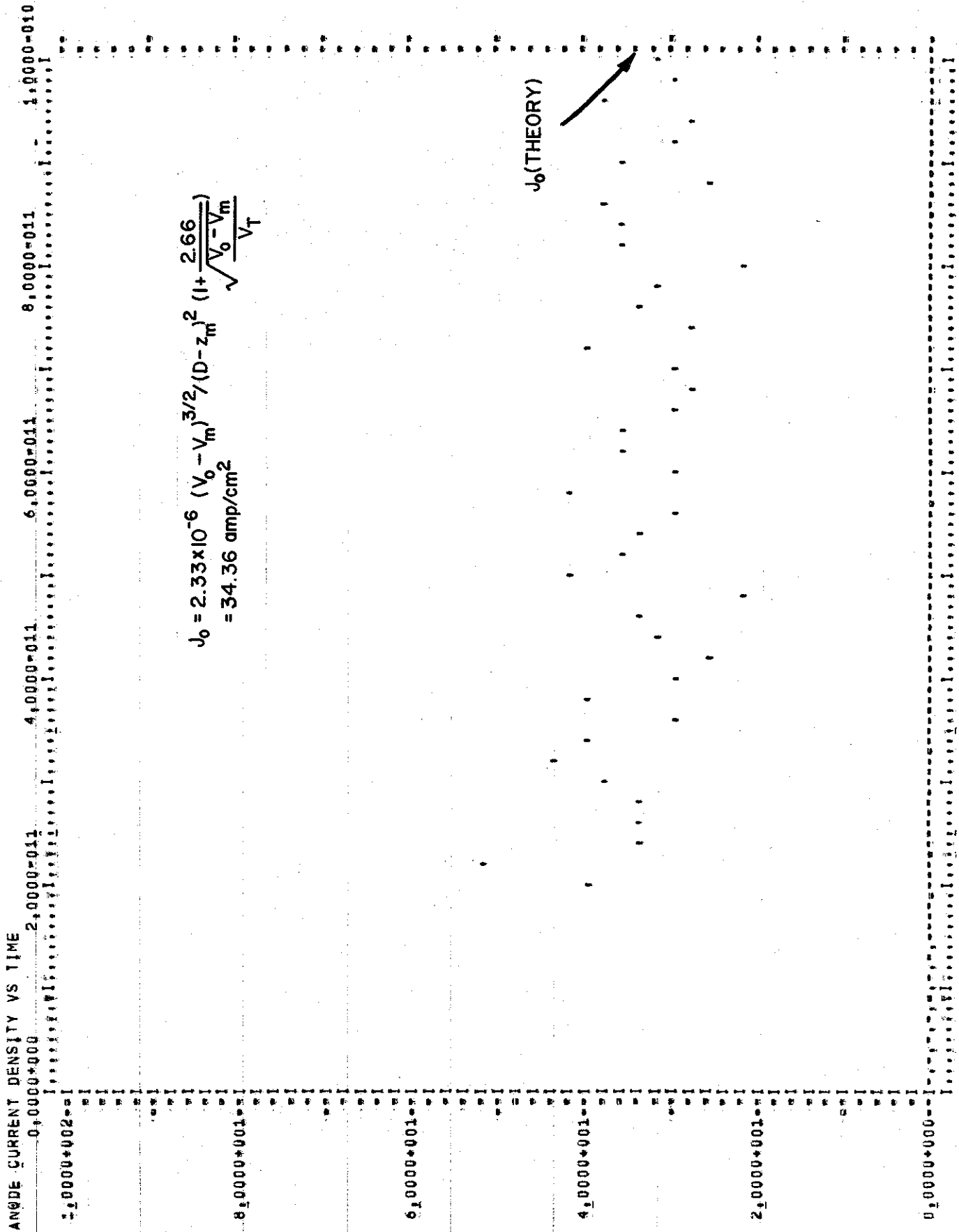


FIGURE 12

DIODE POPULATION VS TIME

Fig 13

0.0000+000 2.0000-011 4.0000-011 6.0000-011 8.0000-011 1.0000-010

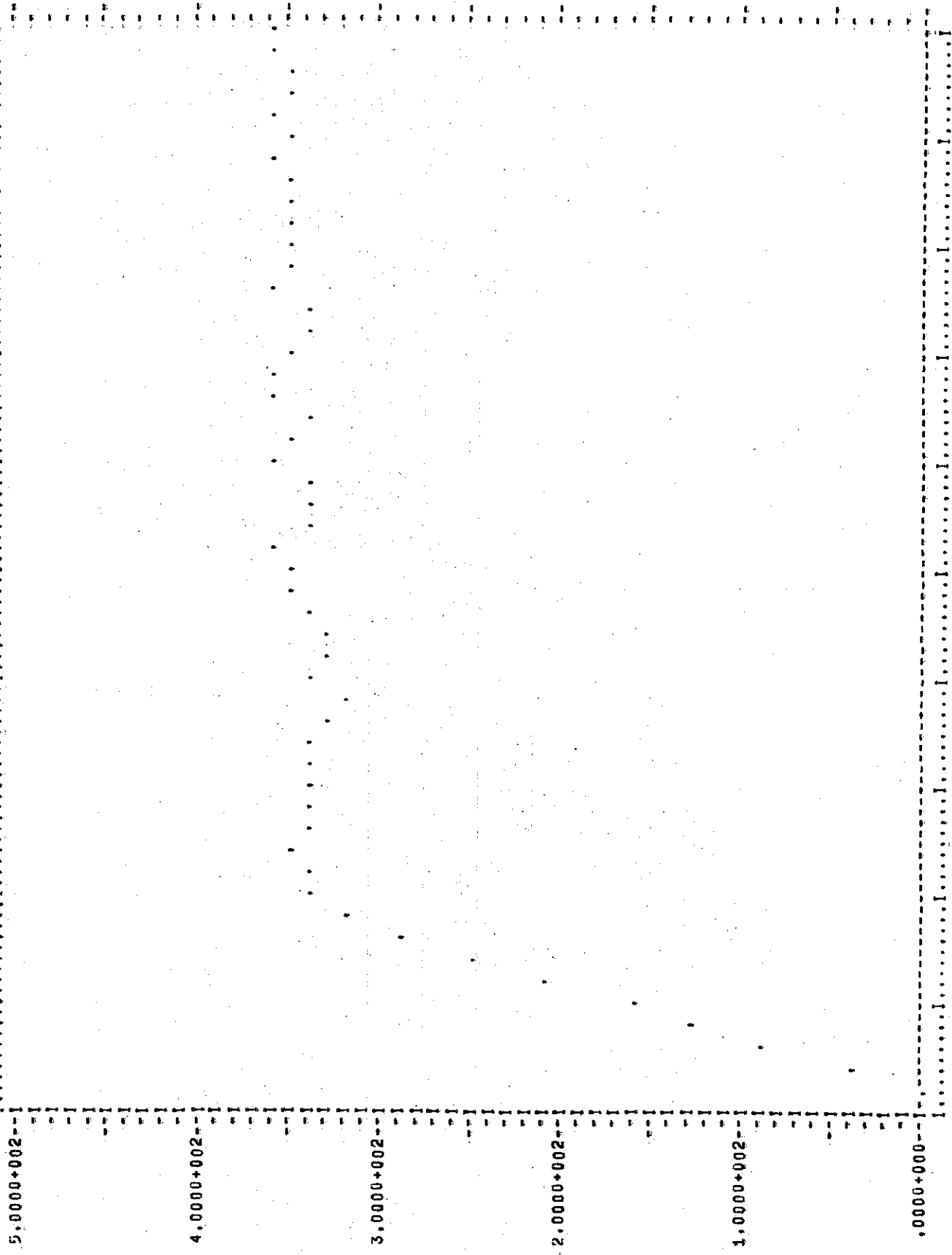


Figure 13

APPENDIX

Program Listing

PROGRAM TOODEE

C*****

C
C
C
C
C
C
C
C
C
C
C
C

TOODEE IS A TWO-DIMENSIONAL TIME-DEPENDENT CODE WHICH SIMULATES THE NON-RELATIVISTIC PARALLEL PLANE DIODE, THE MACROPARTICLES ARE CHARGED RINGS WHICH CAN MOVE Laterally BETWEEN CATHODE AND ANODE, THEY CAN EXPAND AND CONTRACT AS WELL, THE PARTICLES APPEAR AT THE CATHODE PLANE AT RANDOM TIMES, IN RANDOM AMOUNTS WITH A MAXWELLIAN DISTRIBUTION IN R- AND Z-VELOCITY, PARTICLES WHICH REACH THE ANODE OR ARE RETURNED TO THE CATHODE ARE SAVED FOR LATER USE, THIS PERMITS LONG RUNS WITH A MODEST EXPENDITURE OF MEMORY.

C*****

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

DIMENSIONED VARIABLES
 Z,R PARTICLE COORDINATES
 ZP,RP PARTICLE VELOCITIES
 EZS,ERS TWO-DIMENSIONAL TABLES OF FIELDS
 FIRST ARGUMENT=FIELD POINT(MESH POINT)
 SECOND ARGUMENT=SOURCE POINT(WITHIN CELL)
 ER,EZ NET FIELDS AT MESH POINTS
 VECTOR SUM OF SPACE CHARGE AND EXTERNAL FIELDS
 EEXT EXTERNALLY APPLIED FIELD
 ZR STORAGE FOR PARTICLE POSITIONS AT LAST
 ITERATION - USED TO TEST FOR ANODE CROSSING ETC
 BB STORAGE FOR NUMBER OF PARTICLES CROSSING
 ANODE PLANE PER ITERATION
 ZM,RM MESH POINT COORDINATES
 ZS,RS CELL COORDINATES
 X,V USED FOR PLOT OF VOLTAGE VS X
 TT USED IN PLOT OF ANODE CURRENT VS TIME
 CHAR CHARACTER TABLE FOR PLOTTING

COMMON A,B,D,VZERO,VTHERM,DZETA,NSTEP,TSTEP,Q,WEIGHT,T,THETA
 COMMON/ZZ/Z(2500),ZP(2500),R(2500),RP(2500)
 COMMON/FIELD/EZS(64,49),ERS(64,49),ER(64),EZ(64),EEXTZ(8,8),
 IEEXTZ(8,8)
 DIMENSION ZR(2500)
 DIMENSION BB(500),RM(8),RS(7),ZM(8),ZS(7)
 DIMENSION X(11),V(11)
 DIMENSION TT(500)
 DIMENSION CHAR(50)
 DIMENSION PR(500),PJ(50)

C*****

C
C
C
C
C
C
C
C
C
C

CARDS READ
 CARD 1
 VZERO EXTERNAL TUBE VOLTAGE - VOLTS
 D ANODE-CATHODE SPACING - METERS
 VTHERM THERMAL VOLTAGE - VOLTS (CATHODE TEMPERATURE)
 CARD 2

```

C          Q          CHARGE/RING   COULOMBS
C          B          EMISSION RADIUS = METERS
C          WEIGHT     MASS/RING = KILOGRAMS
C
C      CARD 3
C
C          NP          NUMBER OF RINGS AVAILABLE
C          NZ          NUMBER OF CELLS IN THE Z DIRECTION
C          NR          NUMBER OF CELLS IN THE R DIRECTION
C          NSTEP      NUMBER OF ITERATIONS THIS RUN
C          DZETA,TSTEP NUMBER OF SECONDS/ITERATION
C
C      CARD 4
C
C          NCATH      AVERAGE NUMBER OF PARTICLES INJECTED PER STEP
C
C      CARD 5
C
C          NPL0T      NUMBER OF ITERATIONS BETWEEN PLOTS
C
C      CARD 6
C
C          CHAR(I)    CHARACTER TABLE FOR PLOTTING ON PRINTER
C
C      CARD 7
C
C          THETA      ANGLE OF PIERCE ELECTRODE . . . , IN RADIANS
C
C*****
C      READ 80,VZERO,D,VTHERM
80      FORMAT(3E10,3)
C      READ 81,Q,B,WEIGHT
81      FORMAT(4E10,3)
C      READ 82,NP,NZ,NR,NSTEP,DZETA,TSTEP
82      FORMAT(4I5,2E10,5)
C      PRINT 83,VZERO,VTHERM,Q,D,B,WEIGHT,NP,NZ,NR,DZETA,NSTEP,TSTEP
83      FORMAT(7H1VZERO=E12,5/8H VTHERM=E12,5/3H Q=E12,5/
13H D=E12,5/3H B=E12,5/8H WEIGHT=E12,5/4H NP=15/4H NZ=15/
14H NR=15/
27H DZETA=E12,5/7H NSTEP=12/7H TSTEP=E12,5/))
C      READ 85,NCATH
85      FORMAT(I5)
C      PRINT 84,NCATH
84      FORMAT(39H AVERAGE NUMBER OF INJECTED PARTICLES =15)
C      READ 85,NPL0T
C      M=1
C      T=0
C      READ 300,(CHAR(I),I=1,50)
300      FORMAT(50A1)
C      READ 86, THETA
86      FORMAT(E10,3)
C      INITIALIZE PLOT
C      CALL PLOT(1,61,50,,0,)
C      DO 301 I=1,50
301      CALL PLOT(2,I,CHAR(I))
C      A=D
    
```

```

      B=5,*D/7,
      RAVG=0,
      NANODE=0
C     CELL DIMENSIONS, DR BY DZ
      DZ=D/FLOAT(NZ)
      DR=A/FLOAT(NR)
C     SET UP MESH POINTS RM,ZM AND SOURCE POINTS RS,ZS
      NR1=NR+1
      RM(1)=0,
      NZ1=NZ+1
      ZM(1)=0,
      DO 11 J=2,NZ1
11     ZS(J-1)=DZ*FLOAT(J-1)-DZ/2,
      ZM(J)=DZ*FLOAT(J)-DZ
      DO 3000 J=2,NR1
3000    RM(J)=FLOAT(J+1)*DR
      RS(J-1)=SQRT(.5*(RM(J-1)**2.+RM(J)**2.))
      CONTINUE
      PI=3.1415926
C     ASSIGN ALL INITIAL VELOCITIES AND RADIAL POSITIONS
      CALL INITIAL(DZ,NP,NCATH,0)
      MS=0
C     TOTAL NUMBER OF SOURCE POINTS
      NS=NR*NZ
C     TOTAL NUMBER OF FIELD POINTS
      NF=NR1*NZ1
      NF2=NF/2
      PI=3.1415926
C     CALCULATE FIELDS FOR DIODE WITH A PIERCE ELECTRODE
      DO 4050 J=1,NZ1
      DO 4050 K=1,NR1
      IF(RM(K),GT,B) GO TO 4051
      PHI=ATANF(ZM(J)/(B-RM(K)))
      GO TO 4052
4051    PHI=PI-ATANF(ZM(J)/(RM(K)-B))
4052    CONTINUE
      PR=SQRTF(ZM(J)*ZM(J)+(B-RM(K))*(B-RM(K)))
      P=PI/(THETA+PI/2,)
      ARG=ATANF((ZM(J)*COSF(PHI*P)-(B-RM(K))*SINF(PHI*P))/(ZM(J)*
1SINF(PHI*P)+(B-RM(K))*COSF(PHI*P))
      IF(ZM(J),GT,D/2,) ARG=ARG*(D-ZM(J))/(D+ZM(J))
      IF(RM(K),EQ,0,) ARG=0,
      EEXTZ(J,K)=(-VZERO/D)*COSF(ARG)
      EEXTR(J,K)=(-VZERO/D)*SINF(ARG)
      IF(J,EQ,NZ1) EEXTR(J,K)=0,
4050    CONTINUE
      EEXTZ(1,6)=(-VZERO/D)*COSF(PI/4,-THETA/2,)
      EEXTR(1,6)=(VZERO/D)*SINF(PI/4,-THETA/2,)
      EEXTZ(1,7)=(-VZERO/D)*COSF(PI/2,-THETA)
      EEXTZ(1,8)=EEXTZ(1,7)
      EEXTR(1,7)=(VZERO/D)*SINF(PI/2,-THETA)
      EEXTR(1,8)=EEXTR(1,7)
      PRINT 1008
1008    FORMAT(6H EEXTZ)
      PRINT 1010,((EEXTZ(I,J),J=1,NR1),I=1,NZ1)
      PRINT 1009

```

```

1009  FORMAT(6H EEXTR)
      PRINT 1010,((EEXTR(I,J),J=1,NR1),I=1,NZ1)
1010  FORMAT(8(1X,E12.5))
C     SET UP TABLE OF FIELDS AT EACH MESH POINT DUE TO SINGLE CHARGE
C     IN EACH CELL
C     PERMITTIVITY OF FREE SPACE          FARADS/METER
EPS=8.85E-12
CONST=-Q/(4.*PI*PI*EPS)
INDEX=0
DO 101 I1=1,NZ
DO 101 I2=1,NR
MS=MS+1
MF=0
DO 100 I3=1,NZ1
DO 100 I4=1,NR1
MF=MF+1
EZS(MF,MS)=0,
ERS(MF,MS)=0,
ARG1=ATANF((B=RS(I2))/ZS(I1))
ARG2=PI/2*-THETA
IF(ARG1,LT,ARG2) INDEX=1
CALL FIELDS(ZM(I3),ZS(I1),RM(I4),RS(I2),EZ1,ER1,INDEX)
INDEX=0
C     FIELDS ARE IN VOLTS/METER
ER1=ER1*CONST
EZ1=EZ1*CONST*2,
EZS(MF,MS)=EZ1
ERS(MF,MS)=ER1
100  CONTINUE
101  CONTINUE
C     PRINT ARRAY OF FIELDS
DO 20 K=1,NF
PRINT 1011,K
1011  FORMAT(5H ERS(,I2,1H))
PRINT 1010,(ERS(K,J),J=1,NS)
PRINT 1012,K
1012  FORMAT(5H EZS(,I2,1H))
20   PRINT 1010,(EZS(K,J),J=1,NS)
C     INITIALIZE STORAGE FOR TESTING ANODE CURRENT
DO 21 J=1,NP
21   ZR(N)=Z(N)
C     INITIALIZE STORAGE FOR CALCULATING ANODE CURRENT
DO 22 J=1,NSTEP
PR(J)=0,
BB(J)=0,
22  C     NCOUNT WILL BE THE NUMBER OF PARTICLES LOST VIA ANODE OR CATHODE
NCOUNT=0
C     KK WILL BE THE TOTAL NUMBER OF PARTICLES IN THE DIODE AT ANY TIME
KK=0
DO 1560 J=1,50
1560  PJ(J)=0,
DO 1000 KKK=1,NSTEP
C     INJECT IS AN ENTRY POINT IN ROUTINE INITIAL
C     IT PUTS,ON THE AVERAGE, NCATH RINGS AT THE CATHODE PLANE
CALL INJECT(DZ,NP,NCATH,KK)
PRINT 5000,KKK,KK

```

```

5000  FORMAT(215)
C     GET OUT IF PARTICLE SUPPLY IS EXCEEDED
      IF(KK,GE,NP) GO TO 4003
C     ACCEL COUNTS THE PARTICLES IN EACH CELL AND USES THE TABLE
C     OF FIELDS, IT RETURNS WITH THE TOTAL NET FIELD AT EACH MESH POINT
      CALL ACCEL(NZ, NR, KK, NCOUNT, NP, 2, NA)
C     NCOUNT IS THE NUMBER OF RINGS LOST TO CATHODE AND ANODE
      DO 140 N=1, KK
        I=1, +R(N)/DR
        J=1, +Z(N)/DZ
C     INTERPOLATION OF FIELDS WITH RESPECT TO POSITION OF PARTICLE
C     IN EACH CELL
        A1=(R(N)-RM(I))/DR
        B1=1, -A1
        C1=(Z(N)-ZM(J))/DZ
        D1=1, -C1
        L=I+(J-1)*NR1
        M=L+NR1
C     R AND Z ACCELERATIONS
        RPP=D1*(A1*ER(L+1)+B1*ER(L))+C1*(A1*ER(M+1)+B1*ER(M))
        RPP=RPP*(Q/WEIGHT)
        ZPP=D1*(A1*EZ(L+1)+B1*EZ(L))+C1*(A1*EZ(M+1)+B1*EZ(M))
        ZPP=ZPP*(Q/WEIGHT)
C     EULER INTEGRATION TO GIVE NEW POSITIONS AND VELOCITIES
        R(N)=R(N)+DZETA*(RP(N)+0.5*DZETA*RPP)
        Z(N)=Z(N)+DZETA*(ZP(N)+0.5*DZETA*ZPP)
        RP(N)=RP(N)+DZETA*RPP*0.5
        ZP(N)=ZP(N)+DZETA*ZPP*0.5
140   CONTINUE
C     SECOND CALL TO ACCEL UPDATES THE VELOCITIES ONLY
      CALL ACCEL(NZ, NR, KK, NCOUNT, NP, 1, NA)
      KK=KK-NCOUNT
      DO 150 N=1, KK
        I=1, +R(N)/DR
        J=1, +Z(N)/DZ
        A1=(R(N)-RM(I))/DR
        B1=1, -A1
        C1=(Z(N)-ZM(J))/DZ
        D1=1, -C1
        L=I+(J-1)*NR1
        M=L+NR1
        RPP=D1*(A1*ER(L+1)+B1*ER(L))+C1*(A1*ER(M+1)+B1*ER(M))
        RPP=RPP*(Q/WEIGHT)
        ZPP=D1*(A1*EZ(L+1)+B1*EZ(L))+C1*(A1*EZ(M+1)+B1*EZ(M))
        ZPP=ZPP*(Q/WEIGHT)
        RP(N)=RP(N)+DZETA*RPP*0.5
        ZP(N)=ZP(N)+DZETA*ZPP*0.5
C     VELOCITIES NOW CORRECT
150   CONTINUE
C     SKIP OVER N PLOT ITERATIONS BETWEEN PLOTS
      MM=KKK/NPLOT
      NN=MM*NPLOT
      IF(KKK, NE, MM) GO TO 999
C     INITIALIZE VOLTAGES
      DO 4 J=1, 30
        V(J)=0,

```

```

4    CONTINUE
C    POP IS AN ENTRY POINT IN ACCEL * IT PRINTS THE CELL DENSITIES
    CALL POP
C    INTEGRATE THE FIELD ON THE AXIS TO GET THE VOLTAGE
    V(1)=0.
    DO 5 J=1,NZ
    K=J*NR1+1
    V(J+1)=V(J)+EZ(K)*DZ
5    CONTINUE
    DV=V(NZ1)-VZERO
    DO 304 J=2,NZ1
304  V(J)=V(J)-DV
    DO 303 J=1,NZ1
    X(J)=DZ*FLOAT(J-1)
C    VOLTAGE PLOT
303  CALL PLOT(3,4,V(J),X(J))
    PRINT 501,T
501  FORMAT(3H1VOLTAGE ON THE AXIS VS Z AT T=E12.5)
    CALL PLOT(4,0)
C    PLOT OF R VS Z FOR EACH PARTICLE IN DIODE REGION
    DO 4010 J=1,KK
4010  CALL PLOT(3,4,R(J),Z(J))
C    PUT THIS POINT IN ARRAY TO BE PLOTTED TO INSURE THAT WHOLE DIODE
C    APPEARS IN THE PLOT
    X=D
    Y=2*B
    CALL PLOT(3,4,Y,X)
    PRINT 502,T
502  FORMAT(30H1PARTICLE POSITIONS AT TIME T=E12.5)
    CALL PLOT(4,0)
999  CONTINUE
    PR(KKK+1)=KK
    BB(KKK)=NA
    PRINT 305,BB(KKK)
305  FORMAT(4H BB=E10.3)
C    FORM DENSITY VS R
    IF(KKK,LT,10) GO TO 1000
    RR=D/10.
    DO 1500 J=1,KK
    IF(Z(J),LT,D/5.) GO TO 1500
    K=1,+R(J)/RR
    PJ(K)=PJ(K)+10.*Q/(2.*PI*U*R(J))
1500  CONTINUE
C    UPDATE TIME AND RETURN FOR NEXT ITERATION
1000  T=T+TSTEP
    DO 24 J=1,NSTEP
C    MAKE A CURRENT DENSITY IN AMPS/CM**2 USING AVERAGE RADIUS
    BB(J)=-BB(J)+Q/(TSTEP*PI*d*B*10000.)
    TI(J)=TSTEP*FLOAT(J)
C    PLOT ANODE CURRENT DENSITY VS TIME
24   CALL PLOT(3,4,BB(J),TI(J))
    PRINT 500
500  FORMAT(30H1ANODE CURRENT DENSITY VS TIME)
    CALL PLOT(4,0)
    GO TO 4005
4005  PRINT 4004

```

```
4004 FORMAT(39H1NUMBER OF AVAILABLE PARTICLES EXCEEDED)
4005 CONTINUE
C   PLOT OF DIODE POPULATION VS TIME
DO 1501 J=1,NSTEP
1501 CALL PLOT(3,4,PR(J),TT(J))
    PRINT 1502
1502 FORMAT(25H1DIODE POPULATION VS TIME)
    CALL PLOT(4,0)
C   PLOT OF DENSITY VS R
DO 1503 J=1,10
    QJ=J
    TT(J)=QJ*D/10.
1503 CALL PLOT(3,4,FJ(J),TT(J))
    PRINT 1504
1504 FORMAT(31H1DENSITY VS R , ARBITRARY UNITS)
    CALL PLOT(4,0)
    RETURN
END
```


SUBROUTINE INITIAL(DZ, NP, NCATH, KK)

```

C *****
C
C   A CALL TO INITIAL PROVIDES AN INITIAL MAXWELLIAN DISTRIBUTION
C   OF VELOCITIES ZP, RP AND A RANDOM DISTRIBUTION IN R OUT TO R=B.
C   IT ALSO CALCULATES A TABLE, G, WHICH GIVES THE CUMULATIVE
C   DISTRIBUTION FUNCTION FOR AN ASSUMED POISSON DISTRIBUTION OF RINGS
C   EMITTED IN THE INTERVAL TSTEP AT AN AVERAGE OF NCATH PER TSTEP.
C   FOR DETAILS OF THE CALCULATION SEE :
C   TIEN AND MOSHMAN, JAP, 27, 1066, 1956
C *****
C
COMMON/ZZ/Z(2500), ZP(2500), R(2500), RP(2500)
COMMON A, B, D, VZERO, VTHERM, DZETA, NSTEP, TSTEP, Q, WEIGHT, T, THETA
DIMENSION F(150), G(150)
Q=ABSF(Q)
DO 1 J=1, NP
C   RANF PROVIDES A RANDOM NUMBER BETWEEN 0 AND 1
X=RANF(=1)
R(J)=B*SQRTF(X)
X=RANF(=1)
C   CONVERTS A UNIFORM RANDOM DISTRIBUTION TO A MAXWELLIAN DISTRIBUTION
C   FOR THE R- AND Z-VELOCITIES.
K=J/2
K=2*K
IF(K, EQ, J) GO TO 50
SIGN=1,
GO TO 60
50 SIGN=-1,
60 CONTINUE
RP(J)=SQRTF((2, *VTHERM*Q/WEIGHT)*(-LOGF(1, -X)))*SIGN
RP(J)=0,
1 ZP(J)=SQRTF((2, *VTHERM*Q/WEIGHT)*(-LOGF(X)))
SS=1
DO 2 K=1, 150
OK=K-1
K1=K-1
IF(K, EQ, 1) GO TO 35
DO 3 J=1, K1
QJ=J
3 SS=SS+QJ
GO TO 36
35 SS=1,
36 CATH=NCATH
C   POISSON DISTRIBUTION WITH AVERAGE=NCATH
F(K)=EXPF(-CATH)*(CATH**OK)/SS
SS=1
C   CUMULATIVE DISTRIBUTION FUNCTION
DO 5 K=1, 150
4 G(K)=0,
DO 6 J=1, K
6 G(K)=G(K)+F(J)
PRINT 7, G(K)
7 FORMAT(E12,5)

```

```

5   CONTINUE
    G(150)=1,
    Q=-Q
    GO TO 15
    ENTRY INJECT

```

```

C
C*****
C
C   A CALL TO INJECT CALCULATES THE NUMBER OF PARTICLES TO INJECT BY
C   COMPARING A RANDOM NUMBER WITH THE CUMULATIVE DISTRIBUTION FUNC-
C   TION G, IT THEN GIVES THIS NUMBER OF PARTICLES RANDOM POSITIONS
C   OVER THE DISTANCE TSTEP*ZP FOR EACH PARTICLE, THIS SIMULATES
C   RANDOM INJECTION IN TIME.
C*****
C

```

```

    X=RANF(-1)
    DO 10 J=2,150
    IF(X,LT,G(J),AND,X,GE,G(J+1)) GO TO 11
10  CONTINUE
11  KKJ=KK+J
    KK1=KK+1
    PRINT 20,J
20  FORMAT(41H NUMBER OF INJECTED RINGS THIS TIME STEP=15)
    DO 12 M=KK1,KKJ
C   SIMULATE RANDOM TIMES OF INJECTION OF THESE J PARTICLES
12  Z(M)=RANF(-1)*TSTEP*ZP(M)
14  KK=KKJ
15  RETURN
    END

```

SUBROUTINE FIELDS(Z1,Z2,R1,R2,EZ1,ER1,INDEX)

```

C *****
C
C   A CALL TO THIS ROUTINE PROVIDES THE Z-FIELD, EZ1, AND THE R-FIELD
C   ER1, AT POINT Z1,R1 DUE TO RING OF CHARGE AT Z2, RADIUS R2,
C   THE METHOD OF IMAGES TOGETHER WITH THE FREE RING POTENTIAL
C   IS USED TO TAKE ACCOUNT OF THE CONDUCTING PLANES AT Z=0 AND Z=D,
C   20 IMAGES ARE SUFFICIENT FOR EACH CONDUCTOR TO CONVERGE THE SUM
C   OF THE IMAGE FIELDS TO THE CORRECT VALUE FOR ALL DISPOSITIONS
C   OF FIELD POINT - SOURCE POINT IN THE DIODE
C *****
C
COMMON A,B,D,VZERO,VTHERM,DZETA,NSTEP,TSTEP,Q,WEIGHT,T,THETA
PI=3.1415926
ER1=0,
EZ1=0,
IF(R2,GE,B) GO TO 100
RS=(R1+R2)**2,
RD=(ABSF(R1-R2))**2,
NNN=20
DO 10 N=1,NNN
  C   POSITIVE IMAGES AT Z2+2*N*D FOR POSITIVE N
  SIGN=*1
  JJ=1
  QN=N+1
  ZETA=Z1-2,*QN*D-Z2
13  ZETAB=(ABSF(ZETA))**2,
  ALF=4,*R1*R2/(RS+ZETAB)
  V=1,-ALF
  U=ALF*LOGF(V)
  C   COMPLETE ELLIPTIC INTEGRAL OF THE SECOND KIND
  EE=1,+V*(.4630106-0.2452740*U+V*(0.1077857-0.04125321*U))
  C   COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND
  EK=1.38629436-.5*U+V*(0.1119697-0.1213486*U
1  +V*(.07253230-.02887472*U))
  BB=SQRTF(RS+ZETAB)
  ER1=ER1+SIGN*(EK-(R2**2,-R1**2,+ZETAB)*EE/(RD+ZETAB))/(R1*BB)
  EZ1=EZ1+SIGN*ZETA*EE/(BB*(RD+ZETAB))
  GO TO (15,16,17,18),JJ
15  ZETA=Z1-2,*QN*D+Z2
  C   NEGATIVE IMAGES AT 2*N*D-Z2 FOR POSITIVE N
  JJ=2
  SIGN=*1
  GO TO 13
16  QN=N
  IF(N,EQ,NNN) GO TO 10
  C   POSITIVE IMAGES AT Z2+2*N*D FOR NEGATIVE N
  SIGN=*1
  ZETA=Z1+2,*QN*D-Z2
  JJ=3
  GO TO 13
17  SIGN=*1
  C   NEGATIVE IMAGES AT 2*N*D-Z2 FOR NEGATIVE N
  ZETA=Z1+2,*QN*D+Z2

```

```

      JJ=4
      GO TO 13
18     CONTINUE
10     CONTINUE
100    CONTINUE
      IF(INDEX,NE,1) GO TO 11
      BR2=ABSF(B-R2)
      DD=SQRTF((BR2)**2.+Z2**2,)*SINF(THETA+ATANF((B-R2)/Z2))
      ZZ=2, *DD*COSF(PI/2,-THETA)+Z2
      RR=2, *DD*SINF(PI/2,-THETA)+R2
      SIGN=+1
      JJ=1
50     RS=(RR+R1)**2,
      RD=(ABSF(R1-RR))**2,
      ZETA=Z1+ZZ
      ZETAB=(ABSF(ZETA))**2,
      ALF=4, *R1*RR/(RS+ZETAB)
      V=1, *ALF
      U=ALF*LOGF(V)
C      COMPLETE ELLIPTIC INTEGRAL OF THE SECOND KIND
      EE=1, *V*(.4630106-0.2452740*U+V*(0.1077857-0.04125321*U))
C      COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND
      EK=1,38629436-.5*U+V*(0.1119697-0.1213486*U
1      +V*(.07253230-.02887472*U))
      BB=SQRTF(RS+ZETAB)
      ER1=ER1+SIGN*(EK*(RR**2,-R1**2,+ZETAB)*EE/(RD+ZETAB))/(R1*BB)
      EZ1=EZ1+SIGN*ZETA*EE/(BB*(RD+ZETAB))
      GO TO (20,21),JJ
20     JJ=2
      ZZ=2, *D=ZZ
      SIGN=+1
      GO TO 50
21     CONTINUE
11     CONTINUE
      IF(Z1,EQ,0) ER1=0,
      IF(Z1,EQ,D) ER1=0,
      RETURN
      END

```

SUBROUTINE ACCEL(NZ, NR, KK, NCOUNT, NP, NN, NA)

```

C
C*****
C
C   A CALL TO ACCEL WILL CAUSE ALL CELL POPULATIONS TO BE COUNTED,
C   THE FORCES AT THE MESH POINTS WILL THEN BE AUGMENTED BY THE
C   AMOUNT THESE POPULATIONS HAVE CHANGED SINCE THE LAST ITERATION.
C
C*****
C
COMMON /ZZ/ Z(2500), ZP(2500), R(2500), RP(2500)
COMMON /FIELD/ EZS(64,49), ERS(64,49), ER(64), EZ(64), EEXTZ(8,8),
1EEXTR(8,8)
COMMON A, B, D, VZERO, VTERM, DZETA, NSTEP, TSTEP, Q, WEIGHT, T, THETA
DIMENSION LCOUNT(2000)
DIMENSION AA(50)
SIGN=1.
NR1=NR+1
NZ1=NZ+1
C   INITIALIZE FIELDS
DO 10 I=1, NZ1
DO 10 J=1, NR1
K=(I-1)*NR1+J
EZ(K)=EEXTZ(I,J)
ER(K)=EEXTR(I,J)
10 CONTINUE
C   NMN=NUMBER OF CELLS
NMN=NR*NZ
C   INITIALIZE THE POPULATIONS AA
DO 11 K=1, NMN
11 AA(K)=0.
C   CELL SIZES
DZ=D/FLOAT(NZ)
DR=A/FLOAT(NR)
C   NK WILL BE THE NUMBER OF PARTICLES RETURNED TO THE CATHODE
NK=0
C   NA WILL BE THE NUMBER PASSING THE ANODE
NA=0
C   NW WILL BE THE NUMBER HITTING THE WALL
NW=0
C   NPROC WILL BE NUMBER HITTING PIERCE ELECTRODE
NPROC=0
NPP=NP
LK=1
GO TO (400,500)NN
400 CONTINUE
DO 20 I=1, KK
C   COUNT THE PARTICLES IN CELL J,L
L=R(I)/DR+1
J=Z(I)/DZ+1
C   IF PARTICLE I HAS RETURNED TO THE CATHODE, INDEX NK BY 1
IF(Z(I),LE,0) GO TO 21
C   IF PARTICLE I HAS PASSED THE ANODE, INDEX NA BY 1
IF(Z(I),GE,D) GO TO 22
C   IF PARTICLE I HAS A RADIUS GREATER THAN A, INDEX NW BY 1
IF(R(I),GE,A) GO TO 23

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```

C      IF PARTICLE I HAS R LESS THAN ZERO, MAKE R POSITIVE
      IF(R(I),LT,0,0) R(I)=-R(I)
      IF(R(I),LT,0,0) RP(I)=-RP(I)
C      IF PARTICLE I HAS HIT PIERCE ELECTRODE, INDEX NPROC BY 1
      IF(R(I),LT,B) GO TO 401
      PHI=ATANF((R(I)-B)/Z(I))
      IF(PHI.GT,THETA) GO TO 24
401    CONTINUE
      K=(J-1)*NR+L
C      ADD TO POPULATION IN CELL J,L
      AA(K)=AA(K)+1
      GO TO 20
21    NK=NK+1
C      LCOUNT GIVES NUMBER OF THE PARTICLE WHICH HAS BEEN REMOVED
      LCOUNT(LK)=I
      LK=LK+1
      GO TO 20
22    NA=NA+1
C      LCOUNT GIVES NUMBER OF THE PARTICLE WHICH HAS BEEN REMOVED
      LCOUNT(LK)=I
      LK=LK+1
      GO TO 20
23    NW=NW+1
C      LCOUNT GIVES THE NUMBER OF THE PARTICLE WHICH HAS BEEN REMOVED
      LCOUNT(LK)=I
      LK=LK+1
      GO TO 20
24    NPROC=NPROC+1
      LCOUNT(LK)=I
      LK=LK+1
20    CONTINUE
      IMAX=LK
      IF(LK,EQ,1) GO TO 14
      JK=1
13    ISTART=LCOUNT(JK)
      IFINISH=NPP-1
C      FILL IN THE GAPS LEFT BY THE REMOVED PARTICLES AND PUT THE
C      REMOVED PARTICLES AT THE END OF THE LINE AWAITING INJECTION
      DO 12 I=ISTART,IFINISH
      R(I)=R(I+1)
      Z(I)=Z(I+1)
      RP(I)=RP(I+1)
12    ZP(I)=ZP(I+1)
      R(NPP)=RANF(-1)*B
      X=RANF(-1)
      Q=-Q
      RP(NPP)=SQRTF((2.*VTHERM*Q/WEIGHT)*(-LOGF(1,-X)))*SIGN
      RP(NPP)=0,
      SIGN=-SIGN
      ZP(NPP)=SQRTF((2.*VTHERM*Q/WEIGHT)*(-LOGF(X)))
      Q=-Q
      JK=JK+1
      IF(JK,EQ,IMAX) GO TO 14
      DO 50 I=1,IMAX
      LCOUNT(I)=LCOUNT(I)-1
50    CONTINUE

```

```

      GO TO 13
14    CONTINUE
      GO TO 600
      C      WE JUMP TO 500 ON THE SECOND CALL TO ACCEL SINCE HERE ONLY
      C      VELOCITIES ARE AUGMENTED
500   DO 30 I=1, KK
      L=R(I)/DR+1
      J=Z(I)/DZ+1
      K=(J-1)*NR+L
      AA(K)=AA(K)+1
      30   CONTINUE
      IMAX=1
600   CONTINUE
      C      NNN=NUMBER OF CELL POINTS
      NNN=NZ1*NR1
      C      UPDATE FIELDS AT EACH CELL POINT USING NEW POPULATIONS AND
      C      THE ARRAY CONTAINING THE FIELDS DUE TO SINGLE SOURCES
      DO 1 J=1, NNN
      DO 2 K=1, NMN
      EZ(J)=EZ(J)+EZO(J,K)*AA(K)
      ER(J)=ER(J)+ERO(J,K)*AA(K)
      2   CONTINUE
      1   CONTINUE
      GO TO (700,800)NN
700   CONTINUE
      NCOUNT=IMAX-1
      C      PRINT NUMBER OF PARTICLES RETURNED TO CATHODE, NUMBER PASSING
      C      ANODE, NUMBER STRIKING WALL, NUMBER STRIKING PIERCE ELECTRODE,
      C      AND TOTAL LOST.
      PRINT 100, NK, NA, NW, NPROC, NCOUNT
100   FORMAT(2X, 3HNK=15, 2X, 3HNA=15, 2X, 3HNW=15, 2X, 5HNPROC=15, 2X, 6HTOTAL=15
1)
      GO TO 1000
      ENTRY POP
      C      CALL POP TO PRINT DENSITIES OF CELLS
      PRINT 1012, T
1012  FORMAT(23H1CELL DENSITIES AT TIME, E12.5)
      PRINT 1001, (AA(K), K=1, NMN)
1001  FORMAT(10(1X, E11.4))
1000  CONTINUE
800   CONTINUE
      RETURN
      END

```